A HOMOLOGICAL DESCRIPTION FOR THE COLORED JONES POLYNOMIAL

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The Colored Jones polynomials $J_N(q)$ are quantum invariants for links defined from the representation theory of $U_q(sl(2))$. This definition is purely algebraically and combinatorially. For the parameter $N = 2$, it recovers the classical Jones polynomial $J(q)$, which has an alternative definition using skein relations. Bigelow [1] and Lawrence [4] described $J(q)$ as a graded intersection pairing in a covering of a configuration space of the puncture disk, using the Lawrence representation and the skein nature of the invariant for the proof.

Later on, deep connections between representation theory of $U_q(sl(2))$ and homological representations of B_n have been discovered. In 2012, Kohno proved that braid grup representations on highest weight spaces of the Verma modules of $U_q(sl(2))$ are isomorphic to certain specialisations of the Lawrence representations ([3], [2]Corollary4.6). We will describe a modification of the previous theorem which identifies the braid group representations on the heighest weight spaces of the finite dimensional modules of $U_q(sl(2))$. Using this, we will present a construction which will lead towards a homological model for the Colored Jones polynomials, described as a graded intersection pairing in a covering of the configuration space of the punctured disk.

References

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